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Generation of Cluster-Type Entangled Coherent States via Cavity QED

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Abstract In this paper, we present a scheme for generating cluster-type entangled coherent states via cavity QED. The scheme is based on the off-resonant interaction between one atom and N cavities, so the spontaneous emission of the atom can be ignored. The initial states of the N cavities are all prepared in vacuum states. We also discuss the experimental feasibility.

Keywords Cluster state · Coherent states · Ionization detection · Cavity QED

1 Introduction

An important goal of quantum information is to construct a quantum computer. There are two typical models of quantum computation. One is the standard quantum computation which is based on sequences of unitary quantum logic gates [1], the other is entirely different quantum computation which is provided in the form of a specific entangled state (a cluster state [2]) as its resource [3, 4]. In the second quantum computation model, information is written onto cluster state, processed, and read out from the cluster state by one-particle measurements only. Cluster states are not only important resources for quantum computation [3], but also for quantum error correction [5], studies of multiparticle entanglement [6] and fundamental tests of non-locality [7]. So the generation of cluster states have attracted much attention.

Many theoretical schemes of generating cluster states have been proposed in different types of physical systems, such as linear optical systems [8], cavity QED [9–13], ion

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trap [14], superconductor charge qubit [15–17] and semiconductor quantum dot system [18–21]. For example, Zou et al. [8] propose a scheme to generate a four-photon polarization-entangled cluster state by using only linear optical elements and four- photon coincidence detection. They also present another two schemes [12] to generate the cluster states of cavities and atoms in the context of microwave cavity QED. In addition, Ivanov et al. [14] propose an efficient technique for creating cluster states by trapped ions. You et al. [16] propose a scheme for one-step generation of large cluster states using superconducting quantum circuits. These circuits are based on Josephson junctions (JJs) and are regarded as promising candidates of solid-state qubits. And Lin et al. [21] propose an efficient method to generate cluster states in spatially separated double quantum dots with a superconducting transmission line resonator.

Experimentally, Mandel et al. [22] prepare the cluster state of neutral atom in optical lattice. Walther et al. [23] have generated four-photon cluster states and demonstrated the feasibility of the one-way quantum computation. Recently, Lu et al. [24] have generated six-photon cluster state which can be used for quantum computation directly. Su et al. [25] produced the continuous-variable(CV) quadripartite cluster state of the electromagnetic field by utilizing two amplitude-quadrature and two phase-quadrature squeezed states of light and linearly optical transformation. Tokunaga et al. [26] experimentally demonstrate a simple scheme for generating a four-photon entangled cluster state with fidelity over 0.860 ± 0.015. In addition, Kiesel et al. [27] report the experimental detection of a high fidelity four-photon cluster state. Walther et al. [28] also experimentally demonstrate that correlations in a four-qubit linear cluster state cannot be described by local realism.

Up to now, there are several reports about cluster coherent states in cavity QED system. For example, Becerra-Castro et al. [10] propose a scheme to generate a 4-qubit cluster of entangled coherent states in bimodal QED cavities. Their scheme use two bimodal cavities, a single two-level atom and a dispersive atom-field interaction. And the cavities state are initially prepared in coherent states. Jia et al. [13] present an alternative simplified scheme fore generating 4-qubit cluster-type of entangled coherent states. Their scheme is based on resonant interaction of a two-mode cavity with a two-level atom driven by strong classical fields.

In this paper, we present another scheme for generating cluster states of coherent states. Comparing with the schemes of [10] and [13], our scheme is based on off-resonant interaction of one three-level Λ style atom with N vacuum cavities, so the spontaneous emission of atom can be ignored. And in our scheme, initial states of cavities are all prepared in vacuum states, we do not need prepare the initial coherent states. The paper is organized as follows. In Sect. 2, we introduce the three-level atom model. In Sect. 3, we depict the generation of the cluster states of many cavities. In Sect. 4, we show the discussion and conclusions.

2 The Theoretical Model

We consider a three-level atom in Λ configuration crossing a one-mode field cavity. The atomic level configuration is depicted in Fig. 1, where both atomic transitions $|c\rangle \leftrightarrow |g\rangle$ and $|c\rangle \leftrightarrow |e\rangle$ are coupled to the same cavity mode with coupling constants g_1 and g_2. And the same atomic transitions are also driven by two classical fields with Rabi frequencies Ω_2 and Ω_1. The detunings Δ and Δ' are presented in Fig. 1. The cavity field coupling constants g_1 (g_2) and the classical fields' Rabi frequencies Ω_1 (Ω_2) are assumed to be the same for any number of the cavities. The Hamiltonian for the system in Schrödinger picture can be

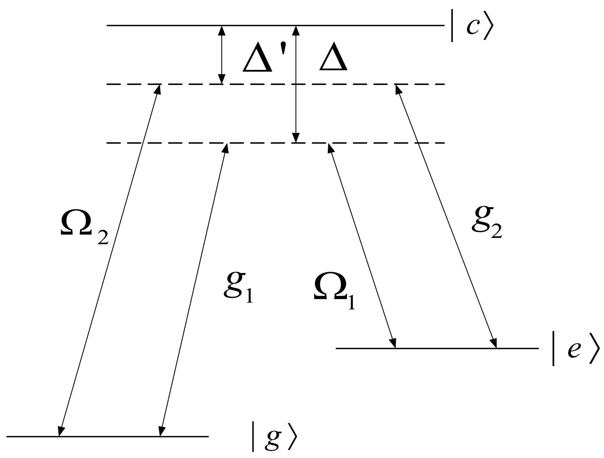

Fig. 1 The diagram of a tree-level atom in Λ configuration. One cavity mode interacts with atomic transitions $|c\rangle \leftrightarrow |g\rangle$ and $|c\rangle \leftrightarrow |e\rangle$ with different constants g_1 and g_2, and the tow classical fields drive the same atomic transitions with Rabi frequencies Ω_2 and Ω_1, respectively. Δ and Δ' are frequency detunings

written as ($\hbar \equiv 1$)

$$H = \omega_c|c\rangle\langle c| + \omega_e|e\rangle\langle e| + \omega a_j^\dagger a_j + (g_1 a_j^\dagger|g\rangle\langle c| + g_2 a_j^\dagger|e\rangle\langle c| + \Omega_1 e^{i\omega_1 t}|e\rangle\langle c| + \Omega_2 e^{i\omega_2 t}|g\rangle\langle c| + H.c), \tag{1}$$

where $a_j^\dagger$ and a_j are the creation and annihilation operators for the jth cavity field with frequency ω, while ω_c and ω_e are the Bohr frequencies associated with the two atomic transitions $|c\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |g\rangle$, respectively, $\omega_1 = \omega_c - \omega_e - \Delta$ and $\omega_2 = \omega_c - \Delta'$. Under the large detuning condition when $\{\frac{g_1}{\Delta}, \frac{\Omega_1}{\Delta}, \frac{g_2}{\Delta'}, \frac{\Omega_2}{\Delta'}\} \ll 1$, the excited level $|c\rangle$ can be eliminated adiabatically. In order to avoid undesired atomic transitions, we need the approximation inequalities $|\Delta - \Delta'| \gg \{\frac{g_1 g_2}{\Delta'}, \frac{\Omega_1 g_2}{\Delta'}, \frac{g_1\Omega_2}{\Delta'}, \frac{\Omega_1\Omega_2}{\Delta'}\}$. So the energy diagram Fig. 1 can be understood as composed by two independent Λ schemes. The Hamiltonian shown in (1) is deduced in interaction picture as follows [29–32]:

$$H_{eff} = -(\lambda a_j^\dagger + \lambda^* a_j)(\sigma^\dagger + \sigma), \tag{2}$$

where $\lambda = \frac{g_1\Omega_1^*}{\Delta} = \frac{g_2\Omega_2^*}{\Delta'}$, $\sigma^\dagger = |e\rangle\langle g|$ and $\sigma = |g\rangle\langle e|$ are the raising and lowering atomic operators, respectively. In (2), we have assumed the compensation of the Stark shifts can be corrected by returning the laser frequencies [33]. We change the atomic bare-state basis into dressed-state basis, i.e.,

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle). \tag{3}$$

After that, the effective Hamiltonian is diagonal as

$$H_{eff} = -(\lambda a_j^\dagger + \lambda^* a_j)(|+\rangle\langle+| - |-\rangle\langle-|). \tag{4}$$

3 Generation of the Cluster States of Many Cavities

In this section, we present a scheme for generating cluster-type entangled coherent states via cavity QED. Now we describe our scheme in detail. Figure 2. shows the setup for the

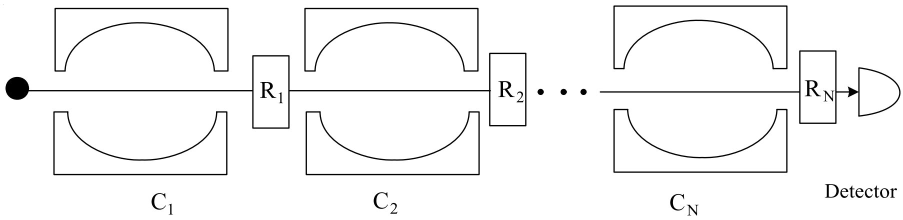

Fig. 2 The setup for generation of cluster-type entangled coherent state. Here a atom is sent through N cavities sequentially. Between every two cavities, N Ramsey zones are employed where the atom is driven by N classical fields, respectively, denoted by R_i $(i = 1, 2, \ldots, N)$. The atom is finally detected by a field ionization detector

generation of cluster-type entangled coherent state. We assume that all cavities are initially prepared in the vacuum state while the atom is in the superposition dressed-state $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$, that is, the initial state of the system is

$$|\Psi\rangle_0 = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)|0\rangle_1|0\rangle_2 \cdots |0\rangle_N. \tag{5}$$

We let the atom enter into the first cavity C_1. The evolution of the system state under the Hamiltonian (4) can be deduced as

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle_1 + |-\rangle|-\alpha\rangle_1)|0\rangle_2 \cdots |0\rangle_N, \tag{6}$$

where $\alpha = i\lambda t$. After leaving the Cavity 1, we let the atom go through the first Ramsey zone R_1 where the atom is driven by the classical field. The interaction will transform at $|+\rangle \to \frac{1}{\sqrt{2}}(|+\rangle| + |-\rangle)$ and $|-\rangle \to \frac{1}{\sqrt{2}}(|+\rangle| - |-\rangle)$. Thus, the (6) becomes

$$\begin{aligned} |\Psi\rangle_1 &= \frac{1}{2}[|+\rangle(|\alpha\rangle_1 + |-\alpha\rangle_1) + |-\rangle(|\alpha\rangle_1 - |-\alpha\rangle_1)]|0\rangle_2 \cdots |0\rangle_N \\ &= \frac{1}{2}[(|+\rangle + |-\rangle\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1)]|0\rangle_2 \cdots |0\rangle_N. \end{aligned} \tag{7}$$

And $\sigma_i = |\alpha\rangle_i\langle\alpha| - |-\alpha\rangle_i\langle-\alpha|$ is performed on the ith cavity mode, here $i = 1$.

Next, the atom goes through the C_2 and R_2, just like the first one, then the state of the total system becomes to

$$\begin{aligned} |\Psi\rangle_2 &= \frac{1}{2\sqrt{2}}(|+\rangle|\alpha\rangle_2 + |-\rangle|-\alpha\rangle_2\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1)|0\rangle_3 \cdots |0\rangle_N \\ &= \frac{1}{2\sqrt{2}}[|+\rangle(|\alpha\rangle_2 + |-\alpha\rangle_2\sigma_1) + |-\rangle(|\alpha\rangle_2 - |-\alpha\rangle_2\sigma_1)] \\ &\quad \otimes (|\alpha\rangle_1 + |-\alpha\rangle_1)|0\rangle_2 \cdots |0\rangle_N \\ &= \frac{1}{2\sqrt{2}}[(|+\rangle + |-\rangle\sigma_2)(|\alpha\rangle_2 + |-\alpha\rangle_2\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1)]|0\rangle_3 \cdots |0\rangle_N. \end{aligned} \tag{8}$$

After that, the atom continues this way, going through C_3, R_3, C_4, $R_3, \ldots, C_N$, R_N, respectively. In this way, the state of the total system becomes to

$$|\Psi\rangle_N = \frac{1}{2^{(N+1)/2}}(|+\rangle + |-\rangle\sigma_N)(|\alpha\rangle_N + |-\alpha\rangle_N\sigma_{N-1})(|\alpha\rangle_{N-1} + |-\alpha\rangle_{N-1}\sigma_{N-2})\cdots(|\alpha\rangle_2 + |-\alpha\rangle_2\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1). \tag{9}$$

We detect the atomic state in the basis $\{|+\rangle, |-\rangle\}$ directly. If the result is $|+\rangle$, the whole system state collapses into

$$|\texttt{Cluster}\rangle_1 = \frac{1}{2^{N/2}}(|\alpha\rangle_N + |-\alpha\rangle_N\sigma_{N-1})(|\alpha\rangle_{N-1} + |-\alpha\rangle_{N-1}\sigma_{N-2})\cdots(|\alpha\rangle_2 + |-\alpha\rangle_2\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1), \tag{10}$$

which is an N-mode cluster-type coherent entangled state. While if the result is $|-\rangle$, the system state becomes

$$|\texttt{Cluster}\rangle_2 = \frac{1}{2^{N/2}}\sigma_N(|\alpha\rangle_N + |-\alpha\rangle_N\sigma_{N-1})(|\alpha\rangle_{N-1} + |-\alpha\rangle_{N-1}\sigma_{N-2})\cdots(|\alpha\rangle_2 + |-\alpha\rangle_2\sigma_1)(|\alpha\rangle_1 + |-\alpha\rangle_1). \tag{11}$$

$|\texttt{Cluster}\rangle_2$ is not the standard cluster state. From the (11), we can see that there is an additional σ_N comparing with the $|\texttt{Cluster}\rangle_1$, that is, the phase of the second cluster state's $|-\alpha\rangle_N$ is opposite to the first cluster state's one. So the success probability of generating the standard cluster state is 50% at least, if we don't like the second cluster state.

4 Discussion and Conclusion

We give a brief discussion on the experimental feasibility of our scheme. In order to generate the cluster state, we need the atom and cavities with long enough lifetimes. Fortunately, in our scheme, the high-level of the atom is eliminated adiabatically, the spontaneous emission can be ignored, so we only consider the lifetime of the cavities. In a recent experiment [34], cavity decay time is about $T_c \sim 10^{-1}$ s. For cluster state, the decoherence time is about $T_d = T_c/2|\alpha|^2 \sim 10^{-2}$ s [13]. We choose $\alpha = i\frac{g_1\Omega_1^*}{\Delta}t = 2$ (when $\alpha = 2$, $|\langle\alpha|-\alpha\rangle|^2 = e^{-4\alpha^2} = 1.1 \times 10^{-7} \simeq 0$), $i\frac{\Omega_1^*}{\Delta} = \frac{1}{100}$, and $g_1 = 2\pi \times 110$ MHz [35], the interaction time between atom and one cavity is $t_i \sim 10^{-7}$ s. The time of the atom crossing one Ramsey zone is about $T_R \sim 10^{-8}$ (when we choose the classical fields' Rabi frequency $\Omega = 10$ MHz in the Ramsey zones). If we prepare a cluster state of ten cavities, the required time is about $T \sim 10 \times (10 \times 10^{-7} + 10 \times 10^{-8}) \sim 10^{-5}$. One notes that the time T is much shorter than the decoherence time of the cluster states. The cluster states can be realized by cavity QED techniques, although it is an experimental challenge to combine these setup together in technique.

In summary, we have proposed a scheme for generating cluster states of cavities. The scheme is based on the off-resonant interaction between the atom and cavities, so the high-level of the atom is eliminated adiabatically. In our scheme, the initial states of the cavities are vacuum states, therefore, we do not need prepare the initial coherent states. We also discuss the quantitatively to experimental feasibility with current technology.

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